Digital Circuits

ECS 371

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Lecture 5-6

Office Hours:
BKD 3601-7
Monday 1:30-3:30
Tuesday 10:30-11:30

ECS371.PRAPUN.COM

Announcement

- I scanned the questions for HW1 and posted them on the course website.
  - Use it if you haven't obtained the textbook.
- Need to do something about the office hours.
- The old office hours
  - Monday 1:30-3:30PM: Conflict with ITS325, MAS210
  - Tuesday 10:30-11:30AM: Conflict with ITS221
- Let's add
  - **Monday 9:00-10:30**
- I'm not limited to these time slots.
  - Usually in my office (BKD3601-7) from 8AM-5PM

Problem Set 1

- Chapter 2

  6, 9, 13, 19, 20, 22, 25, 28
- Chapter 3

  6, 8, 16, 20, 23
- Due date: June 25, 2009 (Thursday)
- Please submit your HW to the instructor 3 minutes BEFORE your class starts.
  - Late submission will not be accepted.
  - Earlier submission is possible. There are two HW boxes in the EC department (6th floor) for ECS 371. (One for CS. Another one for IT.)

Question

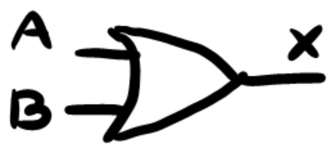
- How many people will participate in the SIIT day activities during our class time on Thursday?

Review

Draw the logic circuit represented by

$X = A \cdot B$

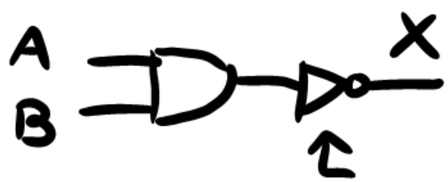
$X = A + B$



$X = A \cdot \overline{B}$

$X = \overline{A \cdot B}$

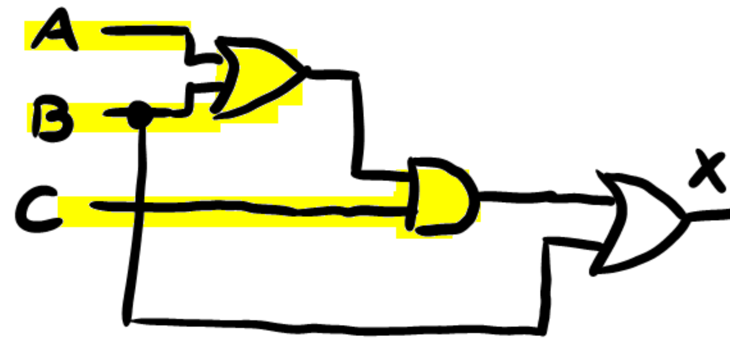
(NAND)



✓

✓

$X = (A + B)C + B$



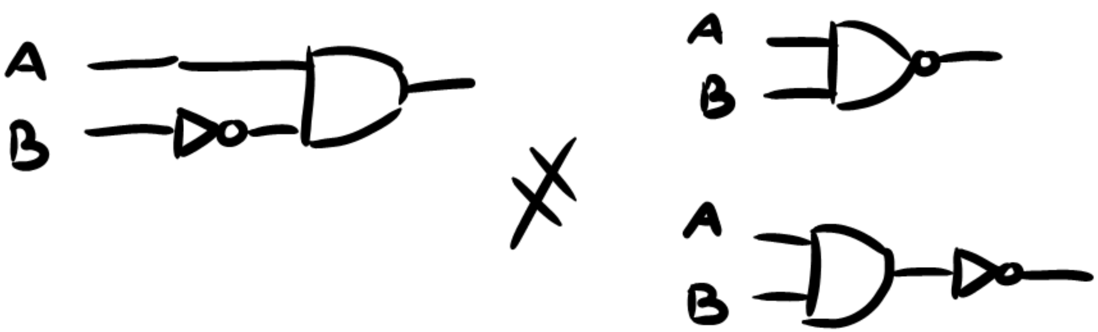
Remark

- Precedence:

$$A + B \cdot \overline{C} \text{ is the same as } A + \left(B \cdot \overline{C}\right)$$

$$A + B \cdot \overline{C} \text{ is NOT the same as } \left(A + B\right) \cdot \overline{C}$$

$$A \cdot \overline{B} \text{ is NOT the same as } \overline{A \cdot B}$$



Truth Table

Example: Find the value of X for all possible values of the variables when

$$X = (A + B) \cdot C + B$$

A	B	C
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

0	0	0
0	0	0
1	0	1
1	1	1
1	0	0
1	1	1
1	0	1
1	1	1
		X

Proving Identities/Rules/Laws

Example: Check that

$$AB + A(B + C) = A(B + C)$$

Method 1: Use Algebraic Manipulation

$$AB + A(B + C)$$
$$= \underbrace{AB + AB}\ + AC$$
$$= A \cdot B + AC$$
$$= A(B + C)$$

#

Method 2: Use Truth Table

A	B	C
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

$AB + A(B+C)$				$A(B+C)$
0	0	0	0	0
0	0	0	1	0
0	0	0	1	0
0	0	0	1	0
0	0	0	0	0
0	1	1	1	1
1	1	1	1	1
1	1	1	1	1

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Comparison

	1 Algebraic Manipulation	2 Truth Table
Advantage	•Quick (usually) ✓ •Simple ✓	•Straightforward ✓
Disadvantage	•Need to remember many rules/laws. ✓ •Need to know when to apply them. ✓	•Tedious/Boring/Time-wasting ✓ ✓ ✓

For ECS371, make sure that you know **both** method.
Later, we will use another method (K-map).

Principle of Duality

Any theorem or identity remains true if $0 \leftrightarrow 1$

$\bullet \leftrightarrow +$

Example:

$$X + 1 = 1 \qquad X + \overline{X} = 1$$

$$X \cdot 0 = 0 \qquad X \cdot \overline{X} = 0$$

Caution: $X + (X \cdot Y) = X \Rightarrow \cancel{X \cdot X + Y = X}$

$X \cdot (X + Y) = X \qquad \cancel{X + Y = X}$

- Parenthesize an expression fully before taking its dual!

$$(X \cdot Y) + (X \cdot Z) = X \cdot (Y + Z)$$

$$(X + Y) \cdot (X + Z) = X + (Y \cdot Z)$$

Duality Principle in Action

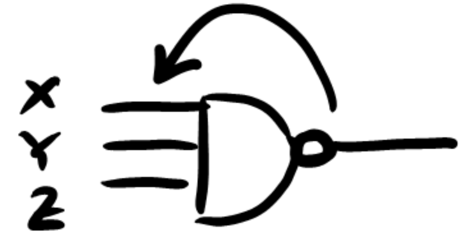
(T1)	$X + 0 = X$	(T1′)	$X \cdot 1 = X$	(Identities)
(T2)	$X + 1 = 1$	(T2′)	$X \cdot 0 = 0$	(Null elements)
(T3)	$X + X = X$	(T3′)	$X \cdot X = X$	(Idempotency)
(T4)	$(X')' = X$			(Involution)
(T5)	$X + X' = 1$	(T5′)	$X \cdot X' = 0$	(Complements)

(T6)	$X + Y = Y + X$	(T6′)	$X \cdot Y = Y \cdot X$	(Commutativity)
(T7)	$(X + Y) + Z = X + (Y + Z)$	(T7′)	$(X \cdot Y) \cdot Z = X \cdot (Y \cdot Z)$	(Associativity)
(T8)	$X \cdot Y + X \cdot Z = X \cdot (Y + Z)$	(T8′)	$(X + Y) \cdot (X + Z) = X + Y \cdot Z$	(Distributivity)
(T9)	$X + X \cdot Y = X$	(T9′)	$X \cdot (X + Y) = X$	(Covering)
(T10)	$X \cdot Y + X \cdot Y' = X$	(T10′)	$(X + Y) \cdot (X + Y') = X$	(Combining)

$$X \cdot Y + X\bar{Y} = X \longrightarrow (X+Y)\cdot(X+\bar{Y}) = X$$

DeMorgan's Theorem

$\overline{X \cdot Y \cdot Z}$



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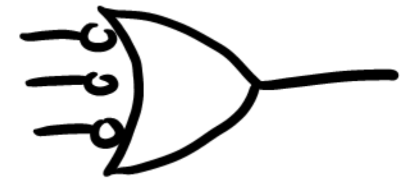


$\bar{X} + \bar{Y} + \bar{Z}$

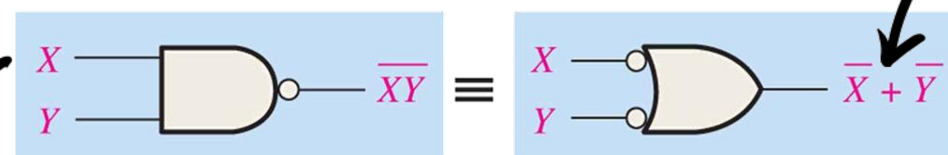
Part 1:

$$\overline{A_1 \cdot A_2 \cdot \cdots \cdot A_n} = \overline{A_1} + \overline{A_2} + \cdots + \overline{A_n}$$

Part 2:

$$\overline{A_1 + A_2 + \cdots + A_n} = \overline{A_1} \cdot \overline{A_2} \cdot \cdots \cdot \overline{A_n}$$

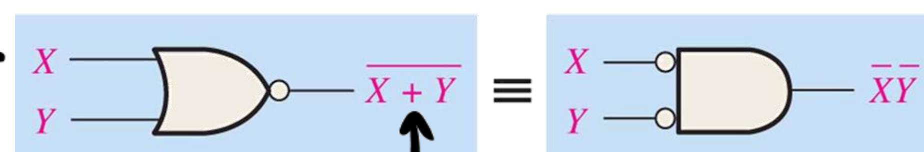
Example:



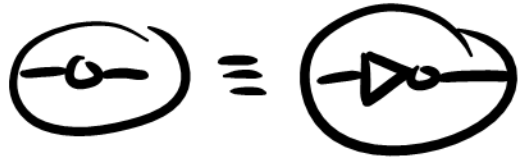
NAND　　　Negative-OR

$$X \text{ NAND } Y = \overline{X \cdot Y} = \overline{X} + \overline{Y} \quad \text{(Negative-OR)}$$

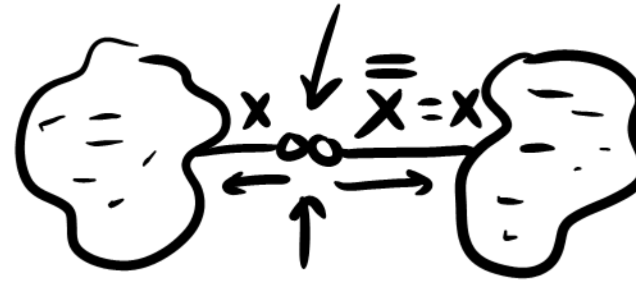
$$X \text{ NOR } Y = \overline{X + Y} = \overline{X} \cdot \overline{Y} \quad \text{(Negative-AND)}$$



NOR　　　Negative-AND



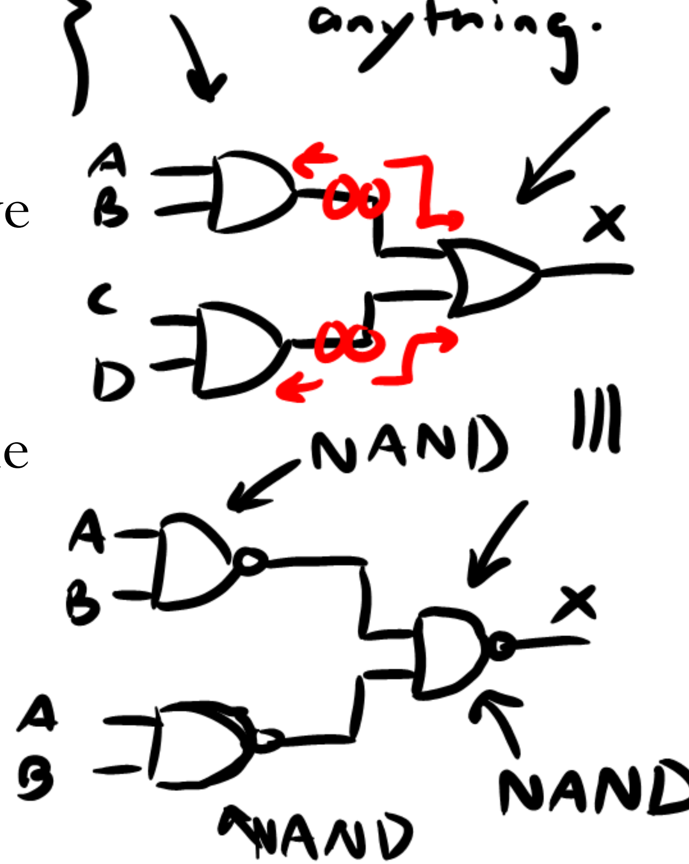
Play with the bubbles



This does not change anything.

Recall that each bubble means a "NOT" operation.

1. You can create a pair of bubbles out of nothing and move them freely on the wire.
2. DeMorgan's Thorem: When you move bubble through the AND gate or the OR gate, the gate changes.
3. If you want to leave an isolated bubble in your final design/answer, write an actual inverter instead of a bubble.



—o— is not a gate

→ —▷o— is a NOT gate

Product Term

A single literal or a product of two or more literals. variable or its complement

Example: $A \cdot \overline{B} \cdot C$ ✓

$A \cdot C$ ✓

A ✓ $\overline{A}$

$A \cdot \overline{B} \cdot C \cdot D$ ✓

$\overline{A} \cdot \overline{B} \cdot \overline{C}$ ✓

Caution:

$\overline{A \cdot B \cdot C}$ is <u>not</u> a product term.

Q: When does $A \cdot \overline{B} \cdot C = 1$?

$A = 1$
$\overline{B} = 1$
$C = 1$

$(A, B, C) = (1, 0, 1)$